WEEKLY TEST TARGET - JEE - TEST - 19

SOLUTION Date 15-09-2019

[PHYSICS]

1. $dQ = du + dW$ or $Q = (u_2 - u_1) + W$

$W = Q_{1b2} - (u_2 - u_1)$

or $\quad Q_{1b2} - W = u_2 - u_1 = 36 - 30 = 6 \text{ cal}$

or $\quad Q_{1b2} - W = u_2 - u_1 = 36 - 30 = 6 \text{ cal}$

2. Work done = Area under the P–V curve

$$W = (80 \text{ kPa})(250 \times 10^{-6}) kt\ 1/2 = 10 \text{ J}$$

Since the arrow is anticlockwise,

$\therefore$ Work done $= -10$ J

3.

$$W = \text{Area } ABCDA = \frac{\pi R^2}{2}$$

$$= \frac{11 \times (20)^2}{2} = 200\ \pi \text{ joule}$$

4.

$$\Delta U = \Delta Q - p\Delta V = \Delta Q - pV$$

5.

Process AB is isochoric, $\quad \therefore W_{AB} = P\Delta V = 0$

Process BC is isothermal $\quad \therefore W_{BC} = RT_2 \cdot \ln\left(\frac{V_2}{V_1}\right)$

Process CA is isobaric

$\therefore W_{CA} = -P\Delta V = -R\Delta T = -R(T_1 - T_2) = R(T_2 - T_1)$

(Negative sign is taken because of compression)

6. .

$$W_{BCOB} = -\text{ Area of triangle } BCO = -\frac{P_0 V_0}{2}$$

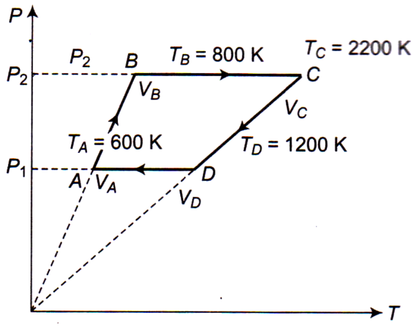
$$W_{AODA} = +\text{ Area of triangle } AOD = +\frac{P_0 V_0}{2}$$

7.

Processes A to B and C to D are parts of straight line graphs of the form $y = mx$

Also $P = \frac{\mu R}{V} T \quad (\mu = 6)$

$\Rightarrow \quad P \propto T$. So volume remains constant for the graphs AB and CD



So no work is done during processes for A to B and C to D i.e., $W_{AB} = W_{CD} = 0$ and $W_{BC} = P_2(V_C - V_B) = \mu R (T_C - T_B)$

$$= 6R(2200 - 800) = 6R \times 1400\ J$$

Also $W_{DA} = P_1(V_A - V_D) = \mu R(T_A - T_B)$

$$= 6R(600 - 1200) = -6R \times 600\ \text{J}$$

Hence work done in complete cycle

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$
$$= 0 + 6R \times 1400 + 0 - 6R \times 600$$
$$= 6R \times 900 = 6 \times 8.3 \times 800 \approx 40\ \text{kJ}$$

8.

$W_{AB} = -P_0V_0$, $W_{BC} = 0$ and $W_{CD} = 4P_0V_0$

$\Rightarrow \quad W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

9.

$$W_{AB} = -\left(P_0V_0 + \frac{P_0V_0}{2}\right) = -\frac{3}{2}P_0V_0$$

$$W_{BC} = (2P_0)(2V_0) + \frac{P_0(2V_0)}{2} = 5P_0V_0$$

$$W_{ABC} = \frac{7}{2}P_0V_0$$

10.

The work done by the gas

$$W = \int_i^f PdV$$

$$W = \int_i^f \alpha V^2 dV = \frac{1}{3}\alpha(V_f^3 - V_i^3)$$

$$V_f = 2V_i = 2(1.00\ \text{m}^3) = 2.00\ \text{m}^3$$

$$W = \frac{1}{3}[(5.00\ \text{atm/m}^6)(1.013 \times 10^5\ \text{Pa/atm})]$$
$$\times [(2.00\ \text{m}^3)^3 - (1.00\ \text{m}^3)^3] = 1.18\ \text{MJ}$$

11. ΔU remains same for both path

For path *iaf*: $\Delta U = \Delta Q - \Delta W = 50 - 20 = 30$ J.

For path *fi*: $\Delta U = -30$ J and $\Delta W = -13$ J

$\Rightarrow \quad \Delta Q = -30 - 13 = -43$ J.

12. **(b)** $\dfrac{m_A C_A}{m_B C_B} = \dfrac{(4/3)\pi r_A^3 \rho_A C_A}{(4/3)\pi r_B^3 \rho_B C_B} = \left(\dfrac{r_A}{r_B}\right)^3 = \dfrac{\rho_A C_A}{\rho_B C_B}$

$$= \left(\frac{1}{2}\right)^3 \times \left(\frac{2}{1}\right) \times \left(\frac{1}{3}\right) = \frac{1}{12}$$

13. **(c)** $\theta_{\text{mix}} = \dfrac{c_W}{m_i + m_W}$

$$\because m_i = m_w \Rightarrow \theta_{\text{mix}} = \frac{\theta_W - \dfrac{L_i}{c_W}}{2} = \frac{80 + 0 - \dfrac{336}{4.2}}{2} = 0°\text{C}$$

14. **(c)** $Q_1 = 10 \times 1 \times 10 = 100$ cal

$Q_2 = 10 \times 0.5(0 - (-20)) + 10 \times 80$

$= (100 + 800)$ cal $= 900$ cal.

As $Q_1 < Q_2$, so ice will not completely melt and final temperature = 0°C.

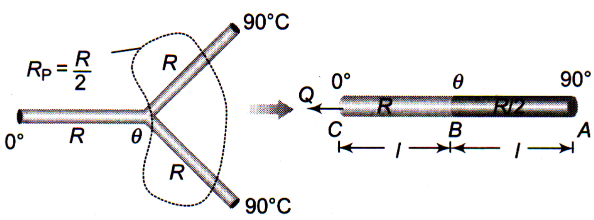
As heat given by water in cooling up to 0°C is only just sufficient to increase the temperature of ice from

15. **(c)** For minimum value of m, the final temperature of the mixture must be 0°C.

$\therefore \quad 20 \times \dfrac{1}{2} \times 10 + 20 \times 80 = m\,540 + m \cdot 1 \cdot 100$

$\therefore \quad m = \dfrac{1700}{640} = \dfrac{85}{32}$ gm

16. **(b)** Let the temperature of junction be θ. Since roads B and C are parallel to each other (because both having the same temperature difference). Hence given figure can be redrawn as follows



$\because \quad \dfrac{Q}{t} = \dfrac{(\theta_1 - \theta_2)}{R}$ and $\left(\dfrac{Q}{t}\right)_{AB} = \left(\dfrac{Q}{t}\right)_{BC}$

$\Rightarrow \quad \dfrac{(90 - \theta)}{R/2} = \dfrac{(\theta - 0)}{R} \Rightarrow 180 - 2\theta = \theta \Rightarrow \theta = 60°\text{C}$

17. **(d)** Radius of cylinder = R and outer radius of shell = nR

$$R_{eq} = \frac{R_1R_2}{R_1+R_2} = \frac{\frac{1}{K_1\pi R^2} \times \frac{1}{K_2(n^2-1)\pi R^2}}{\frac{1}{K_1\pi R^2} + \frac{1}{K_2(n^2-1)\pi R^2}}$$

But $R_{eq} = \dfrac{1}{K\pi n^2R^2}$

On solving, $K = \dfrac{[K_1+K_2(n^2-1)]}{n^2}$

$= \dfrac{4K_1+5K_2}{9}$ (given)

$= \dfrac{K_1+5K_2/4}{9/4}$

$n = 3/2$

18. **(d)** $E = \sigma \times$ area of T^4; T increases by a factor $\dfrac{3}{2}$

Area increase by a factor $\dfrac{1}{4}$

19. **(d)** $Q \propto T^4$

$$\Rightarrow \frac{Q_1}{Q_2} = \frac{T_1^4}{T_2^4} \Rightarrow \frac{E}{E'} = \frac{T^4}{(T/2)^4}$$

which gives $E' = \dfrac{E}{16}$

20. **(d)** Rate of cooling (here it is rate of loss of heat)

$$\frac{dQ}{dt} = (mc+W)\frac{d\theta}{dt} = (m_lc_l + m_cc_c)\frac{d\theta}{dt}$$

$$\Rightarrow \frac{dQ}{dt} = (0.5 \times 2400 + 0.2 \times 900)\left(\frac{60-55}{60}\right)$$

$$= 115\frac{\text{J}}{\text{sec}}.$$

21. **(a)** In first case $\dfrac{61-59}{10} = K\left[\dfrac{61+59}{2} - 30\right]$ (i)

In second case $\dfrac{51-49}{10} = K\left[\dfrac{51+49}{2} - 30\right]$ (ii)

By solving $t = 15$ min.

22. **(d)** According to Newton's law $\dfrac{\theta_1-\theta_2}{t} = k\left[\dfrac{\theta_1+\theta_2}{2} - \theta_0\right]$

Initially, $\dfrac{(80-64)}{5} = K\left(\dfrac{80+64}{2} - \theta_0\right)$

23. **(b)** According to Wien's law, $\lambda_m \propto \frac{1}{T}$

When temperature becomes $\frac{3}{2}$ times λ_m becomes $\frac{2}{3}$ times

24. **(d)** $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\Delta l/l}$

$T = \frac{Y \cdot \Delta l}{l} A = Y \cdot A \alpha \Delta T$

In both the rods tension will be same so

$T_1 = T_2$, Hence $Y_1 A_1 \alpha_1 \Delta T = Y_2 A_2 \alpha_2 \Delta T$

$\frac{A_1}{A_2} = \frac{Y_2 \alpha_2}{Y_1 \alpha_1}$

25. **(b)** Heat given by the sphere

= (1000) (1/2) (80 – 30) = 25,000 cal

Heat absorbed by the water calorimeter system

= (900) (1) (30 – 20) + (200) (1/2) (30 – 20)

= 10,000 cal.

So heat loss to surrounding = 15,000 cal

26. **(b)** Heat conduction through a rod is

$$H = \frac{\Delta Q}{\Delta t} = KA\left(\frac{T_1 - T_2}{l}\right)$$

$\Rightarrow \quad H \propto \frac{r^2}{l}$

When $r = 2r_0$; $l = 2l_0$

$$H \propto \frac{(2r_0)^2}{2l_0} \Rightarrow H \propto \frac{2r_0^2}{l_0}$$

(b) When $r = 2r_0$; $l = 2l_0$

$$H \propto \frac{(2r_0)^2}{l_0} \Rightarrow H \propto \frac{4r_0^2}{l_0}$$

(c) When $r = r_0$; $l = l_0$

$$H \propto \frac{4r_0^2}{l_0}$$

(d) When $r = r_0$; $l = l_0$

$$H \propto \frac{r_0^2}{2l_0}$$

It is obvious that heat conduction will be more in case (b).

Aliter: $\frac{Q}{t} \propto \frac{r^2}{l}$; from the given options, option (b) has higher value of $\frac{r^2}{l}$.

27. **(d)** The product of wavelength corresponding to maximum intensity of radiation and temperature of the body in Kelvin is constant.

According to Wien's law,

$$\lambda_m T = \text{constant (say } b)$$

Where λ_m is wavelength corrsponding to maximum intesity of radiation and T is temperature of the body in Kelvin.

$$\frac{\lambda_{m'}}{\lambda_m} = \frac{T}{T'}$$

Given, $T = 1227 + 273 = 1500$ K,

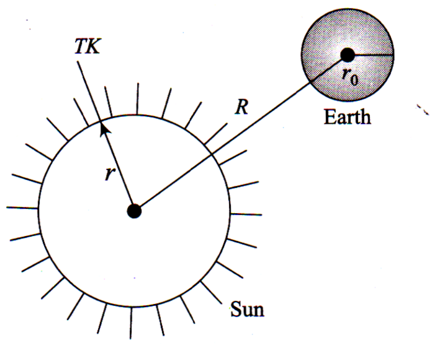
$$T' = 1227 + 1000 + 273 = 2500 \text{ K}$$

$$\lambda_m = 5000 \text{ Å}$$

Hence, $\lambda_{m'} = \frac{1500}{2500} \times 5000 = 3000$ Å

28. **(d)** From Stefan's law, the rate at which energy is radiated by sun at its surface is

$$P = \sigma \times 4\pi r^2 T^4$$



We have taken Sun as a perfect black body as it emits radiations of all wavelengths and so for it $e = 1$.

The intensity of this power at earth's surface (under the assumption $T >> r_0$) is

$$I = \frac{P}{4\pi R^2} = \frac{\sigma \times 4\pi r^2 T^4}{4\pi R^2} = \frac{\sigma r^2 T^4}{R^2}$$

$$\Rightarrow \quad I = \frac{\sigma r^2 (t + 273)^2}{R^2}$$

29. **(c)** The metal X has a higher coefficient of expansion compared to that for metal Y so, on placing bimetallic strip in a cold bath, X will shrink more than Y. Hence, the strip will bend towards the left.

30. **(c)** $\frac{T}{300} = \frac{1}{2}$ or $T = 150$ K

[CHEMISTRY]

31. (c) : Rate $= -\frac{d[N_2O_5]}{dt} = +\frac{1}{2}\frac{d[NO_2]}{dt}$

$= 2\frac{d[O_2]}{dt}$

Given $\frac{-d[N_2O_5]}{dt} = 6{\cdot}25 \times 10^{-3}$ mol L^{-1} s^{-1}

Rate of formation of NO_2

$= \frac{[NO_2]}{dt} = -2\frac{d[N_2O_5]}{dt}$

$= 2 \times 6{\cdot}25 \times 10^{-3}$ mol L^{-1} s^{-1}

$= 12{\cdot}50 \times 10^{-3}$ mol L^{-1} s^{-1}

$= 1{\cdot}25 \times 10^{-2}$ mol L^{-1} s^{-1}

Rate of formation of O_2

$= \frac{d[O_2]}{dt} = -\frac{1}{2}\frac{d[N_2O_5]}{dt}$

$= \frac{1}{2} \times 6{\cdot}25 \times 10^{-3}$ mol L^{-1} s^{-1}

$= 3{\cdot}125 \times 10^{-3}$ mol L^{-1} s^{-1}

32. (b) : Minus signs are for reactants and positive signs for products. Dividing numbers are the coefficients.

33. (b) : $A_2(g) \longrightarrow B(g) + \frac{1}{2}C(g)$

100	0	0
$100 - p$	p	$\frac{1}{2}p$

$100 - p + p + \frac{1}{2}p = 120$ or $p = 40$ mm

$\therefore\ -\frac{dp_{A_2}}{dt} = \frac{40}{5} = 8$ mm min^{-1}

34. (d) : The rate of reaction is same as expressed in terms of any reactant or product.

35. (a) : Rate of reaction $= \frac{1}{4}\frac{\Delta[NO_2]}{\Delta t}$

$= \frac{1}{4} \times \frac{[5{\cdot}2 \times 10^{-3}\,M]}{100\,s} = 1{\cdot}3 \times 10^{-5}$ M s^{-1}.

36. (b) : Rate $= k\,[NO]^2\,[O_2]$. Initially rate $= ka^2 b$. If volume is reduced to half, concentration are doubled so that new rate

$= k\,(2a)^2\,(2b) = 8\,k\,a^2\,b$, *i.e.*, 8 times.

37. (b) : For zero order reaction, $k = \frac{1}{t}\{[A]_0 - [A]\}$

or $[A] = -kt + [A]_0$. Thus, plot of [A] vs t is linear with –ve slope (= $-k$).

38. (c) : From slow step, rate = $k\,[B_2]\,[A]$.

From 1st eqn, $K_{eq} = \frac{[A]^2}{[A_2]}$

or $[A] = \sqrt{K_{eq}[A_2]} = K_{eq}^{1/2}[A_2]^{1/2}$

Hence, rate $= k\,[B_2]\,K_{eq}^{1/2}[A_2]^{1/2}$

$= k'\,[A_2]^{1/2}\,[B_2]$.

Hence, order $= 1\frac{1}{2}$.

39. (c) : On the basis of given units of k, the reaction is of 3rd order.

40. (d) : $r = k\,[A]^{\alpha}\,[B]^{\beta} = k\,a^{\alpha}\,b^{\beta}$. If concentration of B is doubled, $\frac{r}{4} = k\,a^{\alpha}\,(2\,b)^{\beta}$. Dividing 2nd eqn. by 1st eqn.,

$\frac{1}{4} = 2^{\beta}$ or $2\beta = 2^{-2}$. Hence, $\beta = -2$.

41. (a) : As step I is the slowest, hence it is the rate determining step.

42.

43. (d) : $k = \frac{2{\cdot}303}{32}\log\frac{a}{a - 0{\cdot}99\,a}$

$= \frac{2{\cdot}303}{32}\log 10^2 = \frac{2{\cdot}303}{16}\ \text{min}^{-1}$

$t_{99{\cdot}9\%} = \frac{2{\cdot}303}{k}\log\frac{a}{a - 0{\cdot}999\,a}$

$= \frac{2{\cdot}303}{k}\log 10^3 = \frac{3\times 2{\cdot}303}{2{\cdot}303}\times 16$

= 48 min.

44. (d) : $k = \frac{2{\cdot}303}{t}\log\frac{a}{a-x}$

or $\log\frac{a}{a-x} = \frac{kt}{2{\cdot}303} = \frac{2{\cdot}2\times 10^{-5}\times 60\times 90}{2{\cdot}303}$

$= 0{\cdot}0516$.

Hence, $\frac{a}{a-x}$ = antilog 0·0516 = 1·127.

or $\frac{a-x}{a} = 0{\cdot}887$ or $1 - \frac{x}{a} = 0{\cdot}887$

45. (a) : Decrease in concentration from 0·8 M to 0·4 M in 15 minutes means $t_{1/2}$ = 15 minutes. Time taken for decrease in concentration from 0·1 M to 0·25 M means two half-lives, *i.e.*, = 2 × 15 min = 30 min.

46. (c) : $t_{1/2} \propto \frac{1}{a^{n-1}}$ For $n = 2$, $t_{1/2} \propto \frac{1}{a}$

47. (*c*) : At the point of intersection, $[A] = [B]$, *i.e.*, half of the reactant has reacted. Hence, it represents $t_{1/2}$.

48. (*a*) : $[A]$ is kept constant, $[B]$ is doubled, rate is doubled. So rate $\propto [B]$.
$[B]$ is kept constant, $[A]$ is tripled, rate becomes 9 times, so rate $\propto [A]^2$.
Hence, rate law is
$$\text{rate} = k\,[A]^2\,[B]$$

49. (*a*) : $t_{1/2} \propto a^{1-n}$. Hence, $t_{1/2} \propto 1/a^3$ only when $n = 4$.

50. (*b*) : As $k' > k''$, $E_a' < E_a''$ (Greater the rate constant, less in the activation energy).

51. (*d*) : Lower the activation energy, faster is the reaction. Hence, relative ease of P, Q and R will be $R > Q > P$.

52. (*d*) : Activation energy of a particular reaction is constant temperature.

53. (*d*) : % of B $= \dfrac{k_1}{k_1 + k_2}$

$$= \frac{1{\cdot}26\times10^{-4}}{1{\cdot}26\times10^{-4} + 3{\cdot}8\times10^{-5}}\times100$$

$$= \frac{1{\cdot}26\times10^{-4}}{10^{-4}(1{\cdot}26+0{\cdot}38)}\times100 = 76{\cdot}83\%.$$

54. (*c*) : $2\,NH_3 \xrightarrow{Pt} N_2 + 3\,H_2$

$$\text{Rate} = -\frac{1}{2}\frac{d\,[NH_3]}{dt} = \frac{d\,[N_2]}{dt}$$

$$= \frac{1}{3}\frac{d\,[H_2]}{dt} = k = 2{\cdot}5\times10^{-4}\ \text{mol L}^{-1}\,\text{s}^{-1}$$

Rate of production of H_2,

$$\frac{d\,[H_2]}{dt} = 3\times2{\cdot}5\times10^{-4}$$

$$= 7{\cdot}5\times10^{-4}\ \text{mol L}^{-1}\,\text{s}^{-1}$$

55. (*c*) : The reaction occurring in two steps has two activation energy peaks.
The first step, being fast needs less activation energy. The second step, being slow, needs more activation energy. Therefore, second peak will be higher than the first.

56. (*c*) : 10·8 g of N_2O_5 $= \frac{10.8}{108}$ mole = 0·1 mole

No. of half-lives in 9·6 h $= \frac{9.6}{2.4} = 4$

Amount left after 4 half-lives

$$= \frac{1}{2^4} \times 0.1 = \frac{0.1}{16} \text{ mole}$$

Moles of N_2O_5 decomposed

$$= 0.1 - \frac{0.1}{16} = \frac{1.5}{16} \text{ mole}$$

Moles of O_2 formed $= \frac{1}{2} \times \frac{1.5}{16} = \frac{1.5}{32}$

Volume of O_2 at STP $= \frac{1.5}{32} \times 22.4 \text{ L} = 1.05 \text{ L}$

57. (*b*) : $r = K[A]^\alpha = k\,a^\alpha$

$1.837\,r = k\,(1.5\,a)^\alpha$

Dividing, $1.837 = (1.5)^\alpha$

On solving, we get $\alpha = 1.5$

Hence order = 1·5

58. (*c*) : Diagram (*c*) represents enothermic reaction with

$$E_a = 200 - 150 = 50 \text{ kJ and}$$
$$\Delta H = 50 - 150 = -100 \text{ kJ.}$$

59. (*d*) : If $E_a = 0$, $k = A\,e^{-E_a/RT} = A\,e^0 = A$. Hence, k becomes independent of T.

60. (*d*) : Given $\log k = 6 - \frac{2000}{T}$

Comparing with $\log k = \log A - \frac{E_a}{2.303\,RT}$

$$\log A = 6,\ i.e.,\ A = 10^6$$

and $\frac{E_a}{2.303\,R} = 2000$

or $E_a = 2000 \times 2.303 \times 8.314 \text{ J mol}^{-1}$
$= 38294 \text{ J mol}^{-1} \approx 38.3 \text{ kJ mol}^{-1}$

[MATHEMATICS]

61. Slope of the tangent $= \frac{dy}{dx} = \left(\frac{dy}{dt}\right) \Big/ \left(\frac{dx}{dt}\right)$

$= \frac{a(-\sin t + \operatorname{cosec} t)}{a\cos t} = \cot t$

$\left(\text{note that } \frac{d}{dt}(\log\tan(t/2)) = \frac{1}{\tan(t/2)}\sec^2\frac{t}{2}.\frac{1}{2} = \operatorname{cosec}\ t\right)$

62. Given $y = ax^3 + bx^2 + cx$

$\Rightarrow \frac{dy}{dx} = 3ax^2 + 2bx + c.$ At (0, 0), slope of tangent = tan 45º = 1 $\Rightarrow$ c = 1. At (1, 0), slope of tangent = 0 $\Rightarrow$ 3a + 2b + c = 0. Also, when x = 1, y = 0 therefore, 0 = a + b + c.

63. The two curves are $x^2 = u$ .....(1)
and $x = y^2$ .....(2)

From (1), $2x = \frac{dy}{dx}$ and from (2),

$1 = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}\ 1 = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}.$

$\therefore$ Slope of tangent to (1) at (1, 1)
= 2 × 1 = 2
and slope of tangent to (2) at (1, 1)

$= \frac{1}{2\times 1} = \frac{1}{2}.$

If θ is the required angle of intersection, then $\tan\theta = \left|\frac{2-\frac{1}{2}}{1+2\times\frac{1}{2}}\right| = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$

64. Given curve is $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$

diff. w.r.t. x, we get

$n\left(\frac{x}{a}\right)^{n-1}\frac{1}{a} + n\left(\frac{y}{b}\right)^{n-1}\frac{1}{b}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{b}{a}\left(\frac{xb}{ya}\right)^{n-1}$

Hence, slope of tangent at $(a,b) = -\frac{b}{a}$

$\Rightarrow$ Equation of tangent at (a, b) is

$y - b = -\frac{b}{a}(x - a)$

65. The curve $y = be^{-x/a}$ meets y-axis, where x = 0,
i.e., where y = b. So, the given curve meets y-axis at the point (0, b)

Also, $\frac{dy}{dx} = be^{-x/a}\left(-\frac{1}{a}\right)$

$\Rightarrow$ Slope of tangent at (0, b) is $-\frac{b}{a}e^0 = -\frac{b}{a}$

Hence, the equation of tangent at (0, b) is

$$y - b = -\frac{b}{a}(x - 0) \Leftrightarrow ay - ab = -bx$$

$$\Leftrightarrow ay + bx = ab \Leftrightarrow \frac{x}{a} + \frac{y}{b} = 1$$

66. For tangent parallel to X-axis, we should have

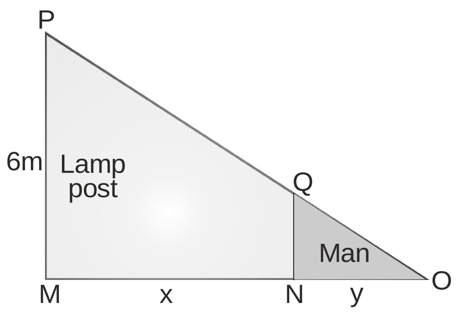
$$\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{d\theta} = 0 \text{ and } \frac{dx}{d\theta} \neq 0$$

$$\Rightarrow \frac{d}{d\theta}\left(3\frac{\sin 2\theta}{2}\right) = 0 \Rightarrow 3\cos 2\theta = 0$$

$$\Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$

67. (1, 1) must lie on both the curves, in particular (1, 1) lies on $ay + x^2 = 7$
Hence, $a \times 1 + 1^2 = 7 \Rightarrow a = 6$

68.



Let at any instant of time t, the distance of man from lamp post be x and the length of his shadow be y, then

$$\frac{6}{x+y} = \frac{2}{y}$$

(By a property of similar triangles)

$\Rightarrow$ $3y = x + y$

$$\Rightarrow y = \frac{1}{2}x, \text{ diff. w.r.t. t, we get}$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{dx}{dt} = \frac{1}{2}(6\,\text{km/hr})$$

$\therefore$ Length of shadow is increasing at the rate of 3 km/hr.

69. Let $f(x) = \log_{10}x$ then $f'(x) = \frac{1}{x\log_e 10}$

Also, $f(x + \Delta x) \simeq f(x) + \Delta x f'(x)$

$$\Rightarrow \log_{10}(x + \Delta x) \simeq \log_{10} x + \frac{\Delta x}{x\log_e 10}$$

Take x = 100 and Δx = – 1 to obtain

$$\log_{10} 99 \simeq \log_{10} 100 + \frac{-1}{100\log_e 10}$$

$$= \log_{10} 10^2 - \frac{1}{100}\log_{10} e$$

$$= 2\log_{10} 10 - \frac{0.4343}{100}$$

= 2 – 0.004343 = 1.995657

70. Let at any instant of time t, the radius of the ice covered spherical ball be r, then volume V at that instant is given by

$$V = \frac{4}{3}\pi r^3$$

Diff. w.r.t., we get

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right)$$

$$\Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} \Rightarrow -100\pi = 4\pi r^2\frac{dr}{dt}$$

($\because$ ice is melting at the rate of 100 π cm^3/min)

$$\Rightarrow \frac{dr}{dt} = \frac{-25}{r^2}$$

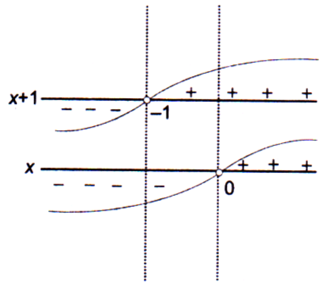
When r = 10 + 5 = 15

($\because$ thickness of ice = 5 cm) then

$$\frac{dr}{dt} = -\frac{25}{(15)^2}\text{cm/min} = -\frac{1}{9}\text{cm/min}$$

Hence, the thickness of ice is decreasing at the rate of $\frac{1}{9}$cm/min

71. For x < 0



f(x) = |x^2 + x| = |(x(x +1)| = x(x + 1)(x < – 1)

f′(x) = 2x + 1

f′(–2) = – 4 + 1 = – 3

$\therefore$ Slope of normal $= \frac{1}{3}$

72. Solving $x^2y = 1 - y$ and $xy = 1 - y$, then (0, 1) and (1, 1, 2)

Now, $x^2y = 1 - y$

$$\Rightarrow \frac{dy}{dx} = -\frac{2xy}{x^2+1}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = 0 \text{ and}$$

$$\left.\frac{dy}{dx}\right|_{(1,1/2)} = -\frac{1}{2}$$

The equations of the required tangent are $y - 1 = 0(x - 0)$ and $y - \frac{1}{2} = -\frac{1}{2}(x - 1)$ y = 1 and x + 2y – 2 = 0

These two tangents intersect at (0, 1).

73. Slope of line (3 – a)x + ay + (a^2 – 1) = 0 is $-\frac{(3-a)}{a}$ or $\left(\frac{a-3}{a}\right)$ ......(i)

$\because$ xy = 1

$$\therefore x\frac{dy}{dx} + y.1 = 0$$

$$\frac{dy}{dx} = -\frac{y}{x} = -\frac{1}{x^2} \qquad (\because xy = 1)$$

$$\therefore \text{Slope of normal} = x^2 = \left(\frac{a-3}{a}\right) \qquad \text{[from eq. (i)]}$$

$\therefore$ $x^2 > 0$

$\frac{a-3}{a} > 0$

$\therefore$ $a \in (-\infty, 0) \cup (3, \infty)$

74. $\because$ $1 \le |\sin| + |\cos x| \le \sqrt{2}$

$\therefore$ y = [|sinx| + |cosx|] = 1

$\because$ $-\sqrt{2} \le \sin x^2 + \cos x \le \sqrt{2}$

$|\sin x| + |\cos x| = \sqrt{|\sin x| + |\cos x|}$

$= \sqrt{(1 + |\sin 2x| \ge 1)}$

Let P and Q be the points of intersection of given curves.

Now, solving y = 1 and $x^2 + y^2 = 5$

$\therefore$ $x^3 + 1 = 5$

$\therefore$ $x = \pm 2$

$\therefore$ $P \equiv (-2, 1)$ and $Q \equiv (2, 1)$

Clearly, the slope of line y = 1 is zero.

$\because$ $x^2 + y^2 = 5$

$\therefore$ $2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow$ $\frac{dy}{dx} = -\frac{x}{y}$

$\left(\frac{dy}{dx}\right)_{(-2,1)} = 2$ and

$\left(\frac{dy}{dx}\right)_{(2,1)} = -2$

Thus, the angle of intersection is $\tan^{-1}(2)$ and $\tan^{-1}(-2)$

75. $\because$ y = f(x)

$\therefore$ $\frac{dy}{dx} = f'(x)$

$\Rightarrow$ $\left.\frac{dy}{dx}\right|_{x=0} = f'(0)$

$\therefore$ Slope of normal $= -\frac{1}{f'(0)} = 3 \Rightarrow f'(0) = -\frac{1}{3}$

Now, $\lim\limits_{x \to 0} \frac{x^2}{\{f(x^2) - 5f(4x^2) + 4f(7x^2)\}}$

Replacing x^2 by x, then

$\lim\limits_{x \to 0} \frac{x}{f(x) - 5f(4x) + 4f(7x)}$

$= \lim\limits_{x \to 0} \frac{x}{\{(f(x) - f(0) - 5(f(4x) - f(0)) + 4(f(7x - f(0))\}}$

$= \lim\limits_{x \to 0} \frac{1}{\left(\frac{f(x)-f(0)}{x-0}\right) - 5\left(\frac{f(4x)-f(0)}{4x-0} \times 4\right)} \cdot \frac{1}{+4\left(\frac{f(7x)-f(0)}{7x-0} \times 7\right)}$

$= \frac{1}{f'(0) - 20f'(0) + 28f'(0)}$

$$= \frac{1}{9f'(0)} \times \frac{1}{9} \times -3 = \frac{-1}{3}$$

76. (b) $by^2 = (x+a)^3 \Rightarrow 2by.\frac{dy}{dx} = 3(x+a)^2 \Rightarrow \frac{dy}{dx} = \frac{3}{2by}(x+a)^2$

$\therefore$ Subnormal $= y\frac{dy}{dx} = \frac{3}{2b}(x+a)^2$

$\therefore$ Subtangent $= \frac{y}{\left(\frac{dy}{dx}\right)} = \frac{y}{\frac{3(x+a)^2}{2by}} = \frac{2by^2}{3(x+a)^2}$

$$= \frac{2b\frac{(x+a)^3}{b}}{3(x+a)^2} = \frac{2}{3}(x+a)$$

$\therefore$ (Subtangent)^2 $= \frac{4}{9}(x+a)^2$

and $\frac{(\text{Subtangent})^2}{\text{Subnormal}} = \frac{\frac{4}{9}(x+a)^2}{\frac{3}{2b}(x+a)^2} = \frac{8b}{27}$

$\Rightarrow$ (Subtangent)^2 = constant $\times$ (Subnormal).

$\therefore$ (Subtangent)$^2 \propto$ (Subnormal).

77. (a) $\sqrt{x} + \sqrt{y} = a$; $\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$, $\therefore$ $\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

Hence tangent at (x, y) is $Y - y = -\frac{\sqrt{y}}{\sqrt{x}}(X - x)$

or $X\sqrt{y} + Y\sqrt{x} = \sqrt{xy}\left(\sqrt{x} + \sqrt{y}\right) = \sqrt{axy}$

or $\frac{X}{\sqrt{a}\sqrt{x}} + \frac{Y}{\sqrt{a}\sqrt{y}} = 1$.

Clearly its intercepts on the axes are $\sqrt{a}\sqrt{x}$ and $\sqrt{a}\sqrt{y}$.

Sum of the intercepts = $\sqrt{a}\left(\sqrt{x} + \sqrt{y}\right) = \sqrt{a}.\sqrt{a} = a$.

78. (b) Clearly the point of intersection of curves is (0, 1). Now, slope of tangent of first curve $m_1 = \frac{dy}{dx} = a^x \log a$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(0,1)} = m_1 = \log a$

Slope of tangent of second curve $m_2 = \frac{dy}{dx} = b^x \log b$

$\Rightarrow m_2 = \left(\frac{dy}{dx}\right)_{(0,1)} = \log b$

$\therefore$ $\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\log a - \log b}{1 + \log a \log b}$.

79. (a) Given $y = 6x - x^2$ .....(i)

$\frac{dy}{dx} = 6 - 2x$

Since, tangent is parallel to the line $4x - 2y - 1 = 0$

$\therefore$ $\frac{dy}{dx} = 6 - 2x = \frac{-4}{-2} \Rightarrow 6 - 2x = 2 \Rightarrow x = 2$

Put the value of x in (i), we get $y = 8$

Hence required point of tangency will be $(2, 8)$.

80. (a) $\frac{dx}{d\theta} = a(1+\cos\theta), \frac{dy}{d\theta} = a(\sin\theta)$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1+\cos\theta)} = 1, \quad y|_{\theta=\frac{\pi}{2}} = a$$

Length of sub-tangent $ST = \frac{y}{dy/dx} = \frac{a}{1} = a.$

and length of sub-normal $SN = y\frac{dy}{dx} = a.1 = a$

Hence $ST = SN$.

81. (c) $x|_{\theta=\frac{\pi}{4}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}},$

$$y|_{\theta=\frac{\pi}{4}} = \frac{3}{2\sqrt{2}}, \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} \left.\frac{9\sin^2\theta\cos\theta}{-6\cos^2\theta\sin\theta}\right|_{\theta=\frac{\pi}{4}} = \frac{-3}{2}.$$

$\therefore$ Equation of tangent is $\left(y - \frac{3}{2\sqrt{2}}\right) = \frac{-3}{2}\left(x - \frac{1}{\sqrt{2}}\right)$

$\Rightarrow 3\sqrt{2}x + 2\sqrt{2}y = 6 \Rightarrow 3x + 2y = 3\sqrt{2}$.

82. (b) Curve $x + y = e^{xy}$

Differentiating with respect to x

$$1 + \frac{dy}{dx} = e^{xy}\left(y + x\frac{dy}{dx}\right) \text{ or } \frac{dy}{dx} = \frac{ye^{xy} - 1}{1 - xe^{xy}}$$

$$\frac{dy}{dx} = \infty \Rightarrow 1 - xe^{xy} = 0 \Rightarrow 1 - x(x+y) = 0$$

This hold for $x = 1$, $y = 0$.

83. (b) We have $f(x)g(x) = 1$

Differentiating with respect to x, we get

$f'g + fg' = 0$ .....(i)

Differentiating (i) *w.r.t.* x, we get

$f''g + 2f'g' + fg'' = 0$ .....(ii)

Differentiating (ii) *w.r.t.* x, we get

$$f'''g + g'''f + 3f''g' + 3g''f' = 0$$

$$\Rightarrow \frac{f'''}{f'}(f'g) + \frac{g'''}{g'}(fg') + \frac{3f''}{f}(fg') + \frac{3g''}{g}(gf') = 0$$

$$\Rightarrow \left(\frac{f'''}{f'} + \frac{3g''}{g}\right)(f'g) = -\left(\frac{g'''}{g'} + \frac{3f''}{f}\right)(fg')$$

$$\Rightarrow -\left(\frac{f'''}{f'} + \frac{3g''}{g}\right)(fg') = -\left(\frac{g'''}{g'} + \frac{3f''}{g}\right)fg', \text{ [using (i)]}$$

$$\Rightarrow \frac{f'''}{f'} + \frac{3g''}{g} = \frac{g'''}{g'} + \frac{3f''}{f} \Rightarrow \frac{f'''}{f'} - \frac{g'''}{g'} = 3\left(\frac{f''}{f} - \frac{g''}{g}\right).$$

84. (d) $I_n = \frac{d^{n-1}}{dx^{n-1}}[x^{n-1} + nx^{n-1}\log x]$

$I_n = (n-1)! + nI_{n-1} \Rightarrow I_n - nI_{n-1} = (n-1)!$.

85. (a) Given that $x = \sin t$, $y = \sin pt$

$\frac{dx}{dt} = \cos t$, $\frac{dy}{dt} = p\cos pt$

$\therefore \frac{dy}{dx} = \frac{p\cos pt}{\cos t} = \frac{p\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Again differentiate $w.r.t.\ x$,

$$\frac{d^2y}{dx^2} = \frac{p\sqrt{1-x^2}.\frac{1}{2\sqrt{1-y^2}}.(-2y)\frac{dy}{dx} - p\sqrt{1-y^2}.\frac{1}{2\sqrt{1-x^2}}(-2x)}{(1-x^2)}$$

$(1-x^2)\frac{d^2y}{dx^2} = -py\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}\frac{dy}{dx} + px\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

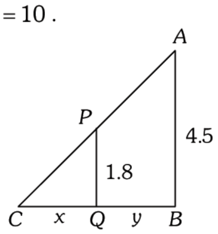
$(1-x^2)\frac{d^2y}{dx^2} = -p^2y + x\frac{dy}{dx}$

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

86. (c) Here $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{40}{4\pi r^2} = \frac{5}{32\pi}$

$\therefore \frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi \times 8 \times \frac{5}{32\pi} = 10$.



87. (a) Given curve $y^2 = px^3 + q$ .....(i)

Differentiate with respect to x, $2y.\frac{dy}{dx} = 3px^2$

$\Rightarrow \frac{dy}{dx} = \frac{3p}{2}\left(\frac{x^2}{y}\right)$

$\therefore \left|\frac{dy}{dx}\right|_{2,3} = \frac{3p}{2} \times \frac{4}{3} = 2p$

For given line, slope of tangent $= 4$

$\therefore 2p = 4 \Rightarrow p = 2$

From equation (i), $9 = 2 \times 8 + q \Rightarrow q = -7$.

88. (a) $y=\frac{2}{3}x^3+\frac{1}{2}x^2 \Rightarrow \frac{dy}{dx}=2x^2+x$ .....(i)

Now tangent makes equal angle with axis

$\therefore\ y=45^o$ or -45^o

$\therefore\ \frac{dy}{dx}=\tan(\pm 45^o)=\pm\tan(45^o)=\pm 1$

$\therefore$ From equation (i), $2x^2+x=1$ (taking $+ve$ sign)

$\Rightarrow 2x^2+x-1=0 \Rightarrow (2x-1)(x+1)=0$

$\therefore x=\frac{1}{2},-1$

From the given curve, when $x=\frac{1}{2}$, $y=\frac{2}{3}.\frac{1}{8}+\frac{1}{2}.\frac{1}{4}=\frac{1}{12}+\frac{1}{8}=\frac{5}{24}$ and when $x=-1$, $y=\frac{2}{3}(-1)+\frac{1}{2}.1=$ $-\frac{2}{3}+\frac{1}{2}=-\frac{1}{6}$

Therefore, required points are $\left(\frac{1}{2},\frac{5}{24}\right)$ and $\left(-1,-\frac{1}{6}\right)$.

89. (d) Slope of the normal $=\frac{-1}{dy/dx}$

$\Rightarrow\ \tan\frac{3\pi}{4}=\frac{-1}{(dy/dx)_{(3,4)}}$

$\therefore\ \left(\frac{dy}{dx}\right)_{(3,4)}=1$, $\therefore\ f'(3)=1$.

90. (d) $y^3+3x^2=12y \Rightarrow 3y^2.\frac{dy}{dx}+6x=12.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(3y^2-12)+6x=0 \Rightarrow \frac{dy}{dx}=\frac{6x}{12-3y^2}$

$\Rightarrow\ \frac{dx}{dy}=\frac{12-3y^2}{6x}$

Since tangent is parallel to y-axis

$\therefore\ \frac{dx}{dy}=0 \Rightarrow 12-3y^2=0$ or $y=\pm 2$.

Then $x=\pm\frac{4}{\sqrt{3}}$. At $\left(\pm\frac{4}{\sqrt{3}},-2\right)$; the equation of curve doesn't satisfy.